

Spectroscopic version of the Aharonov-Bohm effect

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Abstract

An experiment is proposed in which the Aharonov-Bohm effect can be verified through a spectroscopic measurement. The apparatus consists of gaseous hydrochloric acid (HCl) immersed in the constant vector potential $\mathbf{A} = A_0 \mathbf{z}$ present in the interior of a toroidal coil. Changes due to \mathbf{A} in the absorption spectrum of the gas are investigated.

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Introduction and conclusions

The Aharonov-Bohm effect [1] consists in an observable influence of the potentials A^μ acting on a quantum system ψ in a region where $\vec{E} = \vec{B} = 0$. A very simple apparatus in which this effect should be observable is illustrated in Fig. 1.

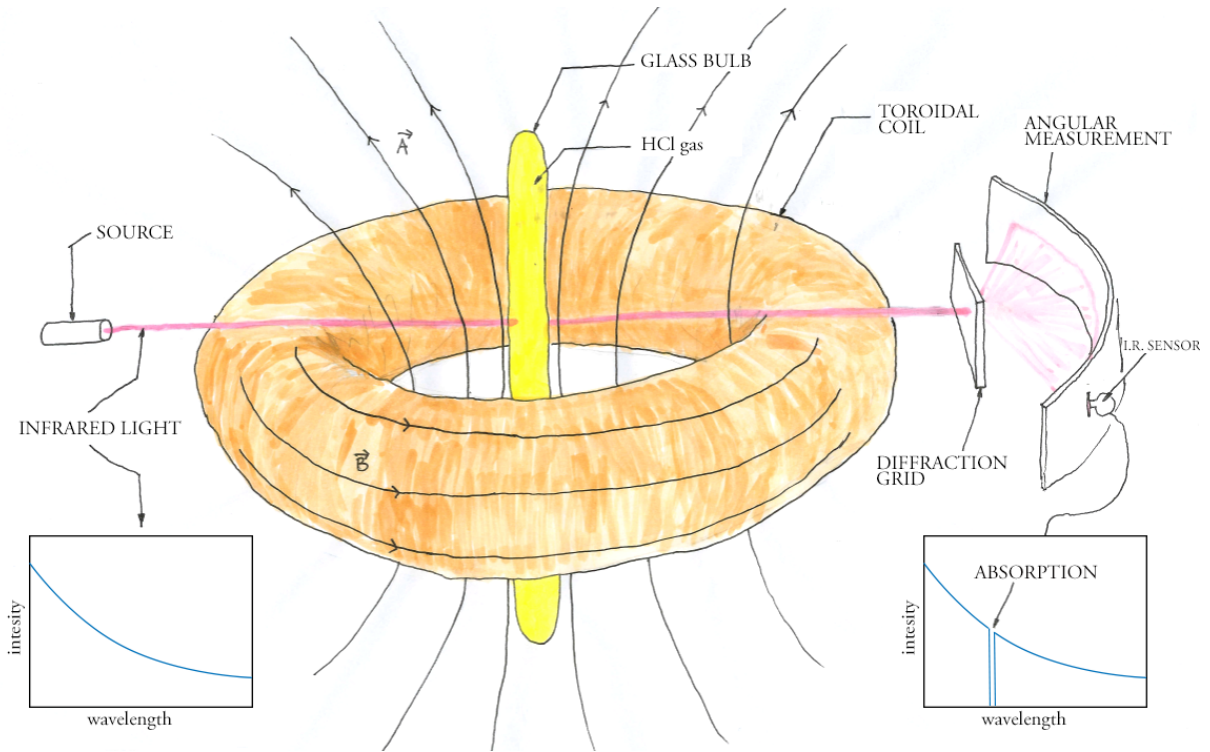


Fig. 1: Experimental apparatus proposed to observe the Aharonov-Bohm effect.

The toroidal coil provides, around its exterior, a region with zero magnetic field but non-vanishing vector potential, onto which the infrared absorption spectrum of gaseous hydrochloric acid is measured.

The vibrational¹ spectrum of HCl is well described by the one-dimensional quantum harmonic oscillator Hamiltonian,

$$\mathcal{H} = \frac{1}{2\mu} [p^2 + (\mu\omega_0 x)^2] \quad (1)$$

where $\mu = \frac{m_{\text{H}}m_{\text{Cl}}}{m_{\text{H}}+m_{\text{Cl}}}$ and ω_0 is the characteristic frequency of vibration around equilibrium. p and x represents the relative momentum and position between hydrogen and chlorine. In the presence of a constant vector potential $\mathbf{A} = A_0\mathbf{z}$, the HCl Hamiltonian reads

$$\mathcal{H} = \frac{1}{2\mu} [p^2 - 2eA_0p \cos(\theta) + (\mu\omega_0 x)^2] \quad (2)$$

where θ is the angle between the molecule axis and \mathbf{A} . Only the component of \mathbf{A} parallel to the molecule axis was used in the minimal coupling,

$$p \rightarrow p - eA_0 \cos(\theta). \quad (3)$$

The matricial representation of (2) in the usual basis $\{|n\rangle\}$ reads,

$$\begin{aligned} \mathcal{H} &= \hbar\omega_0(a^\dagger a + \frac{1}{2}) - ieA_0c\sqrt{\frac{\hbar\omega_0}{2\mu c^2}} \cos(\theta)(a^\dagger - a) \\ &= \begin{pmatrix} \frac{1}{2}\hbar\omega_0 & i\alpha & 0 & 0 & 0 \\ -i\alpha & \frac{3}{2}\hbar\omega_0 & i\alpha\sqrt{2} & 0 & 0 \\ 0 & -i\alpha\sqrt{2} & \frac{5}{2}\hbar\omega_0 & i\alpha\sqrt{3} & 0 \\ 0 & 0 & -i\alpha\sqrt{3} & \frac{7}{2}\hbar\omega_0 & \dots \\ 0 & 0 & 0 & \vdots & \ddots \end{pmatrix} \end{aligned} \quad (4)$$

where

$$\alpha = eA_0c\sqrt{\frac{\hbar\omega_0}{2\mu c^2}} \cos(\theta). \quad (5)$$

Assuming the system has access only to the first two levels of energy, the truncation

$$\mathcal{H} = \begin{pmatrix} \frac{1}{2}\hbar\omega_0 & i\alpha \\ -i\alpha & \frac{3}{2}\hbar\omega_0 \end{pmatrix} \quad (6)$$

leads to the eigenvalues

$$E_{\pm} = \hbar\omega_0 \left[1 \pm \sqrt{\frac{1}{4} + \left(\frac{\alpha}{\hbar\omega_0} \right)^2} \right]. \quad (7)$$

Note that if $\alpha = 0$ the original energy levels $\frac{1}{2}\hbar\omega_0$ and $\frac{3}{2}\hbar\omega_0$ are recovered.

¹The context allows to neglect rotational energy levels, which are closely separated as compared to vibration.

The Aharonov-Bohm effect can be verified by observing an enlargement of the HCl absorption lines when the coil is turned on. Particularly, the first absorption, originally a sharp line located at $\frac{\Delta E}{\hbar} = \omega_0$, would range from

$$\frac{\Delta E}{\hbar} = \omega_0, \quad (8)$$

corresponding to light absorbed by molecules orthogonal to \vec{A} , up to

$$\frac{\Delta E}{\hbar} = 2\omega_0 \sqrt{\frac{1}{4} + \left(\frac{eA_0c}{\sqrt{2\hbar\omega_0\mu c^2}} \right)^2}, \quad (9)$$

corresponding to light absorbed by molecules parallel to \vec{A} .

As a final remark, we explicitly quote the expression of \vec{A} along the z-axis of the torus [2], in the approximation of inner radius (a) much smaller than revolution radius (b),

$$A_z = \frac{\mu_0}{4\pi} \frac{\pi a^2 b I}{(b^2 + z^2)^{\frac{3}{2}}}. \quad (10)$$

For a torus with dimensions $a = 2$ cm and $b = 6$ cm and 10^3 wire loops, a current of order ~ 1.35 A is necessary in order to

$$\frac{eA_0c}{\sqrt{2\hbar\omega_0\mu c^2}} \sim 1, \quad (11)$$

thus making the enlargement of the absorption lines observable.

Discussion

For a long time, the interpretation of A^μ as being a real entity have been the subject of controversy due to the non gauge-invariance of such an object. The thought experiment originally idealized by Bohm and Aharonov consisted in measuring an interference pattern between two branches of a spitted coherent electron beam that traversed the outside region of a long solenoid, where $\vec{E} = \vec{B} = 0$ but $\vec{A} \neq 0$. At the time the experiment was performed and the interference pattern observed, many objections were made based on the fact that the residual field of the non-ideal solenoid could be “touching” the electron, thus transmitting the phase shift. Future experiments with toroidal-shaped devices confined by superconductor material [3] led to the confirmation in favour of the “orthodoxial” point of view: the potentials A^μ are indeed fields².

Despite the fact that the Aharonov-Bohm effect is, nowadays, a paradigm of quantum mechanics, it is worth pointing out an alternative method for verifying the reality [4] of \vec{A} . One advantage of the method suggested here is the usage of a macroscopical source for producing \vec{A} , in contrast to the μm -scale devices used in the diffractive versions of the effect. Also the technologies involved in the proposed apparatus date back to the 19th century.

²The word “field” as synonym of something transmitting interaction. Interaction~Reality.

References

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